17/11/2015 المحاضرة 7

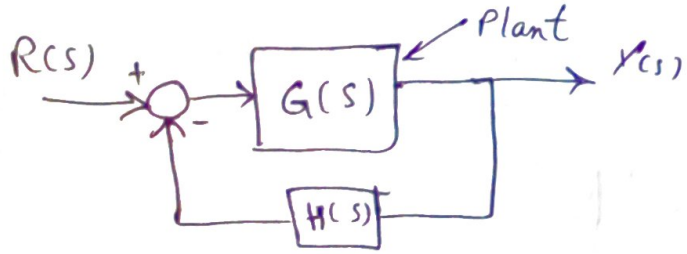
Start Point ⇒ $\omega = 0$ → Real $\angle 0^\circ$ ⇒ type 0
→ $\infty \angle -90^\circ$ ⇒ type 1
→ $\infty \angle -180^\circ$ ⇒ type 2

end point ⇒ $\omega = \infty$ → $|(n_p - n_z) * (-90)|$ for any type
origin

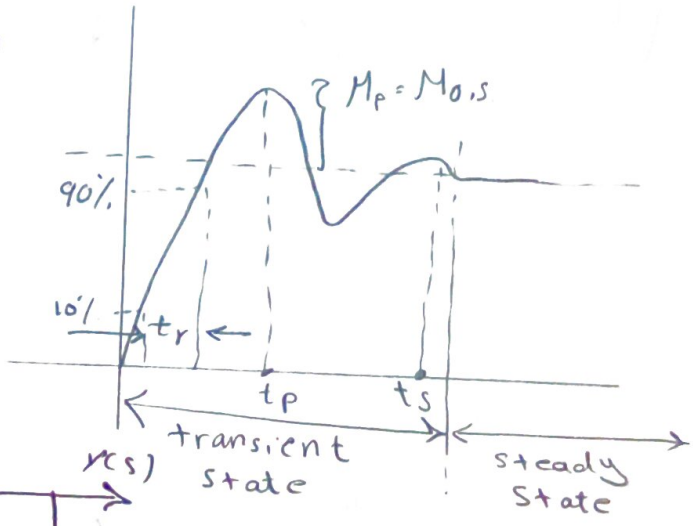
Design of Controllers :- (Compensator)

The controllers are used to improve the system performance → System dynamics (overshoot, t_r, t_p, ...)
→ steady state error

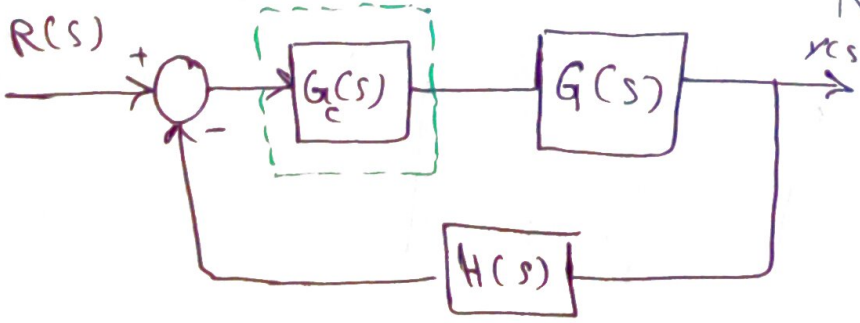
Common structures of Controllers :-



uncompensated System



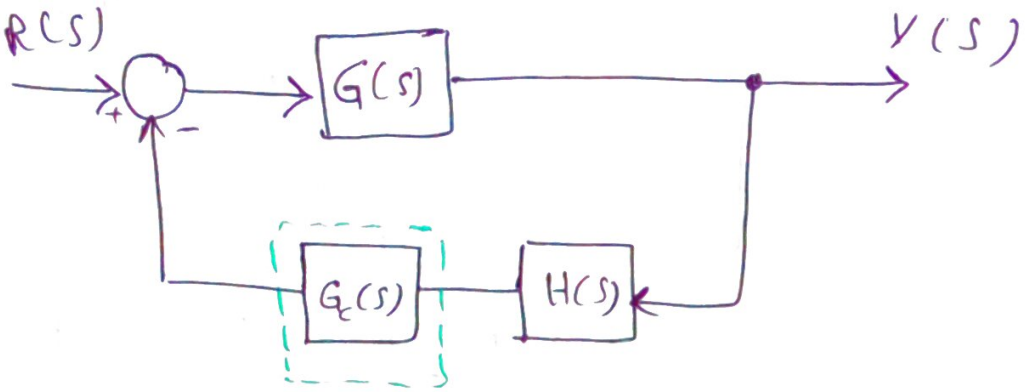
1



Compensated System

← Structure من أنواع مستخدم

2



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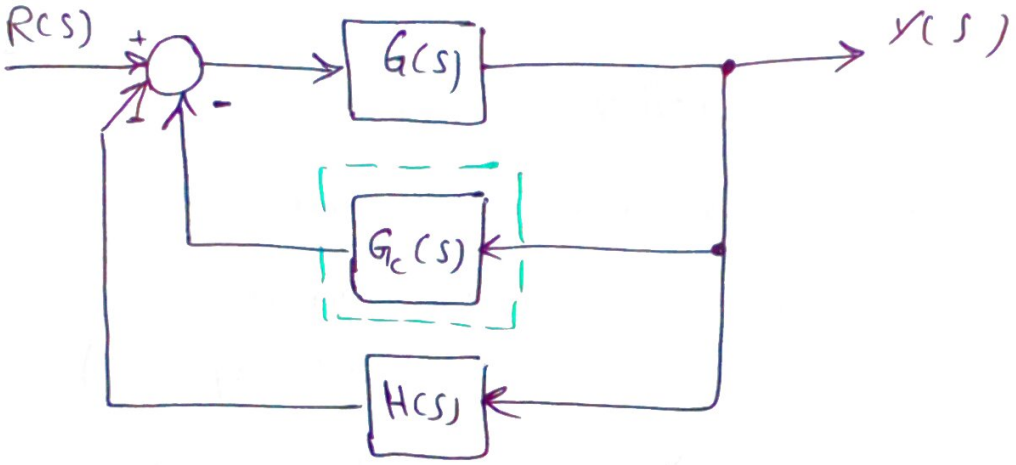

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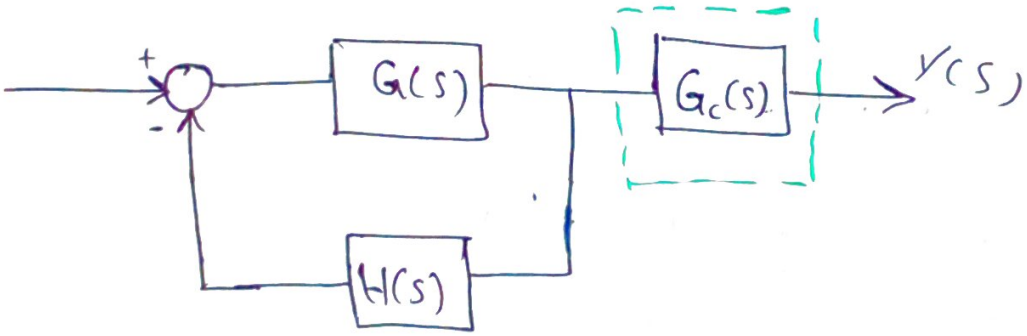

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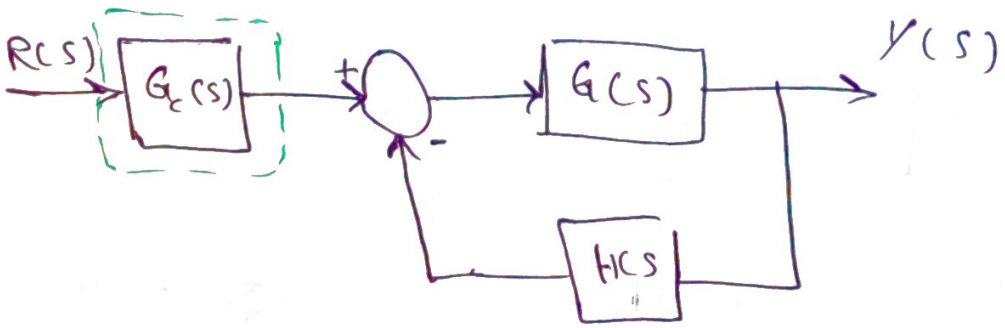

Structure (1) is the most used one

Controllers → ① Software Controllers
Controllers → ② H/W Controllers

Classification of Controllers

1. Classical Controllers / traditional Controllers
   - PI Controllers ⇒ (improve the steady state error)
   - PD Controllers ⇒ (improve system dynamics ⇒ speed up the system response and reduce overshoot)
   - PID
   - phase-Lead Controller ⇒ (The same as PD controller)
   - " - Lag Controller ⇒ (" " " PI ")
   - " - Lead-Lag Controller ⇒ (" " " PID ")

2. Modern Controllers
   - Polar Placement design / state-feedback control design
   - State estimator / observer design

⇒ Based on the states of the system

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

⟹ Turn Over

[3] Computational Intelligent Controllers

The controllers are built with the help of

1. Fuzzy Control (FC)
2. Neural network (NN) ⇒ (Neural + fuzzy)
3. Adaptive Neuro fuzzy inference system (ANFIS)

[4] Evolutionary Computation (Population based)

- non inspired ⇒
  - → Differential evolution (DE)
  - → Partical Swarm optimization (PSO)
- inspired from Nature
  - → Genatic Algorithm (GA)
  - → Artificial Bee conlony Algorithm (ABC)

Remember: for PID controller

$$T.F. = \boxed{K_p + K_d s + \frac{K_I}{s}}$$

$$G_{c_{PID}} =$$

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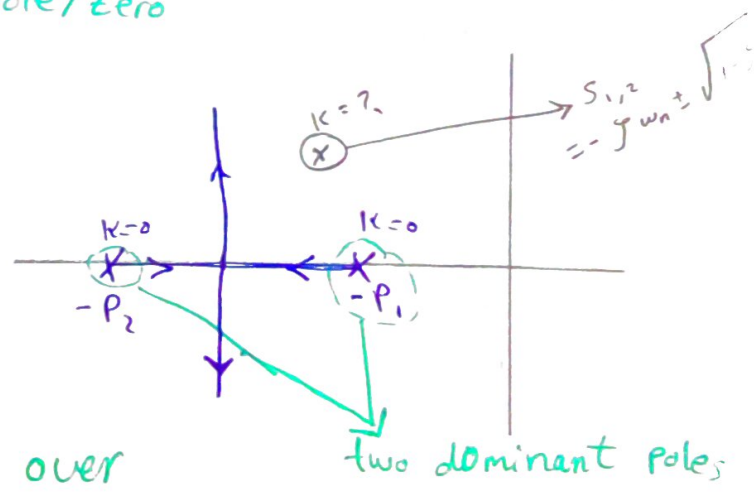
* phase-Lead Controller

Effect of Adding pole/zero

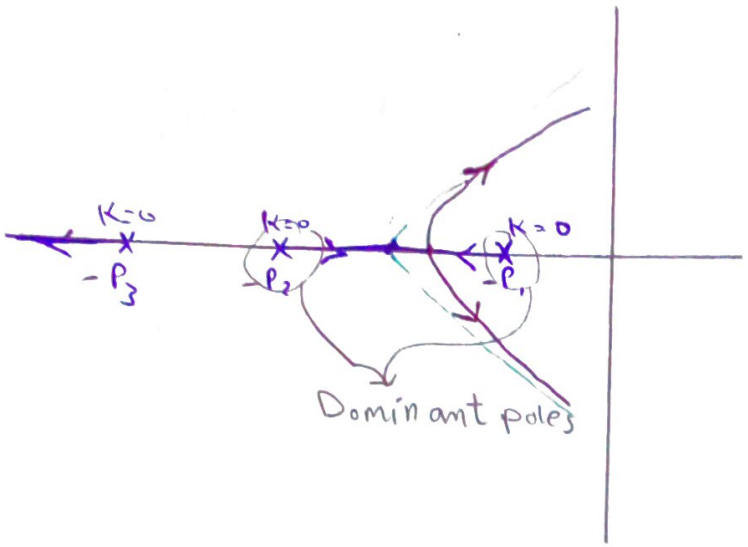
Ex:

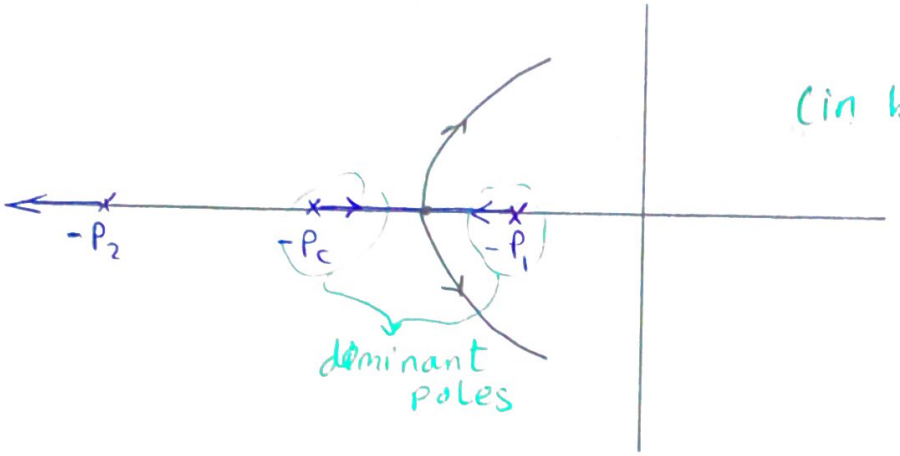
$$GH(s) = \frac{k}{(s+P_1)(s+P_2)}$$



⇒ turn over

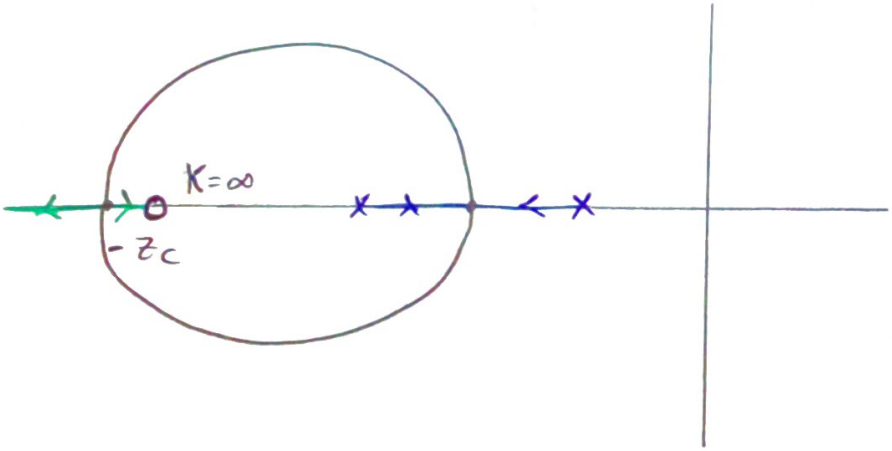
* By Adding Pole (P)



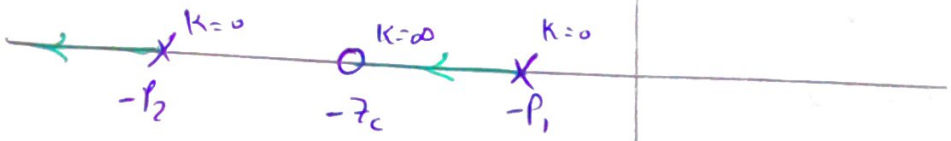


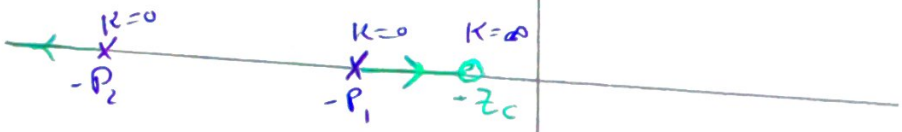
(in between)

* By Adding Zero (Z)



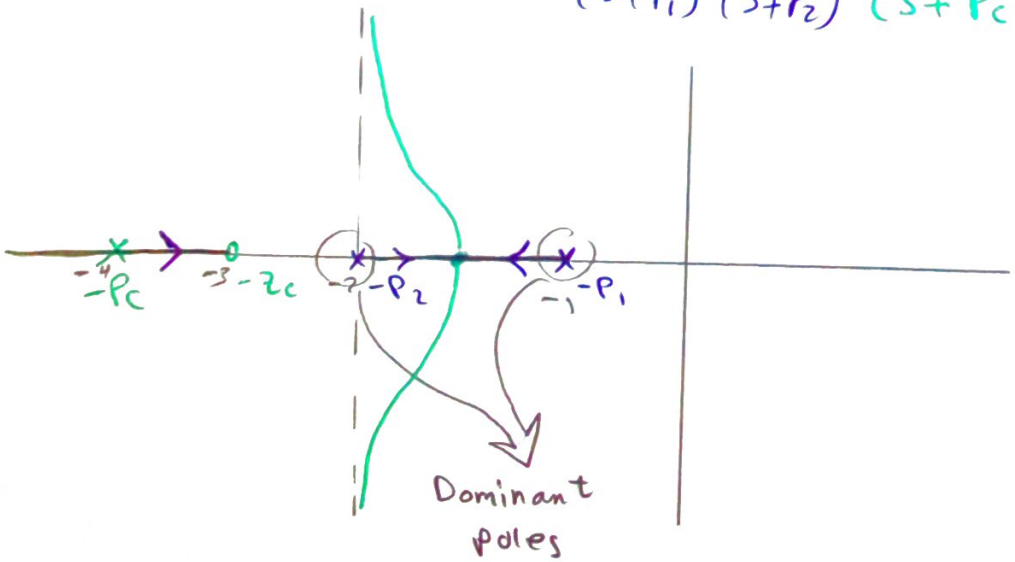
(in between)

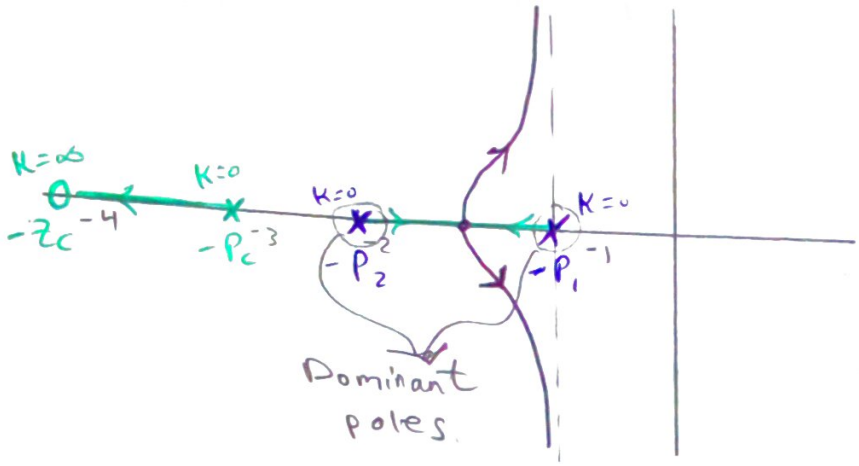




* By Adding pole (P) and zero (Z)

$$GH(S) = \frac{K\ (S+Z_c)}{(S+P_1)(S+P_2)(S+P_c)}$$



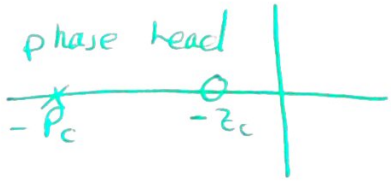


* $G_c(s) = \dfrac{s + z_c}{s + p_c}$

one zero at $-z_c$

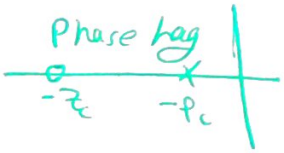
\ pole \ $-p_c$

$|z_c| < |p_c| \Rightarrow$ phase lead



improve System dynamics

$|p_c| < |z_c| \Rightarrow$ phase lag

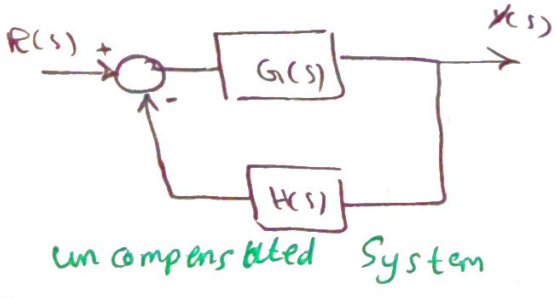


improve steady state error

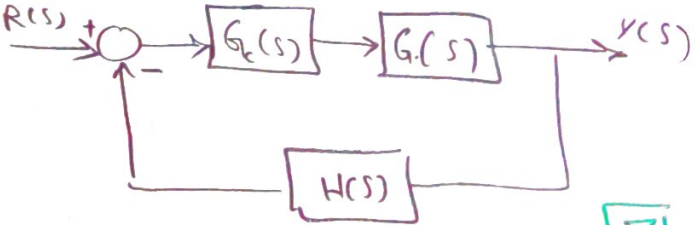
$G_c(s) = \dfrac{s + z_c}{s + p_c}$ (Phase_lead Controller)

$|z_c| < |p_c|$

The requirement is to find the location of z_c, p_c that meet the required design spec



uncompensated System

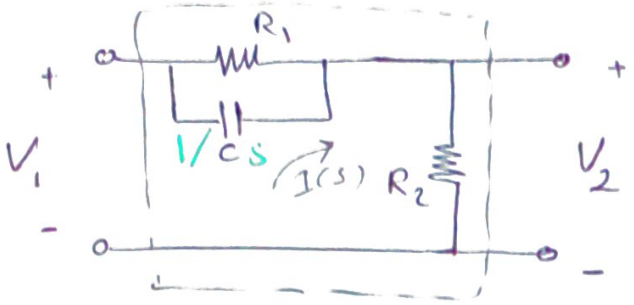


O.L.T.F = $G H(s)$ (Uncompensated System)

Total O.L.T.F = $G_c(s) \cdot G H(s)$

(Compensated System)

For Electrical system



$$G_c(s) = \frac{s + z_c}{s + p_c}$$

$$G_c(s) = \frac{V_2(s)}{V_1(s)} = \frac{I(s) \cdot R_2}{I(s) \cdot Z_{eq}} \quad (2)$$

$$Z_{eq} = \left[R_1 // \frac{1}{cs} \right] + R_2 = \frac{R_1}{R_1 cs + 1} + R_2$$

(2) $$\frac{R_1 \left(\frac{1}{cs}\right)}{R_1 + \frac{1}{cs}} = \frac{R_1}{R_1 cs + 1} \quad (1)$$

$$G_c(s) = \frac{R_2}{\frac{R_1}{R_1 cs + 1} + R_2} = \frac{R_2 (R_1 cs + 1)}{R_1 + R_2 R_1 cs + R_2}$$

$$= \frac{R_2 R_1 cs + R_2}{R_1 R_2 cs + R_1 + R_2}$$

$$= \frac{s + \frac{1}{R_1 c}}{s + \frac{R_1 + R_2}{R_1 R_2 c}}$$

$$G_c(s) = \frac{s + z_c}{s + p_c} = \frac{s + 1/R_1 c}{s + \frac{R_1 + R_2}{R_1 R_2 c}}$$

$$z_c = \frac{1}{R_1 c}, \quad p_c = \frac{R_1 + R_2}{R_1 R_2 c}$$

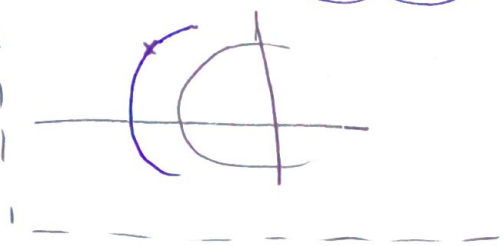
$$G_c(s) = \frac{s + z_c}{s + p_c} = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$$

* The design steps to find z_c, p_c

① Using the given required specs ($\zeta, \omega_n, t_r, M_p$ ⇒ system dynamics) to obtain the location of the desired ples.

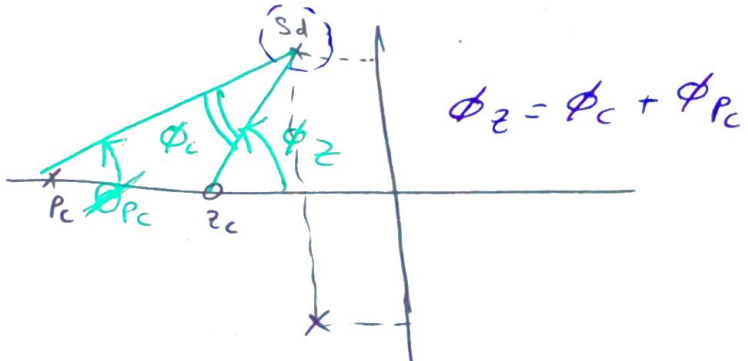
$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \Rightarrow s_{d1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

② Apply the angle condition to obtain the compensator angle (ϕ_c)



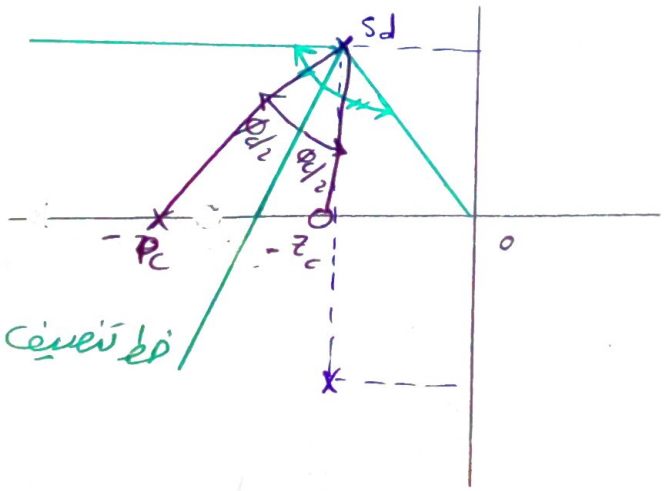
$$\angle GH + \phi_c = -180$$

$$\phi_c = \phi_z - \phi_p$$

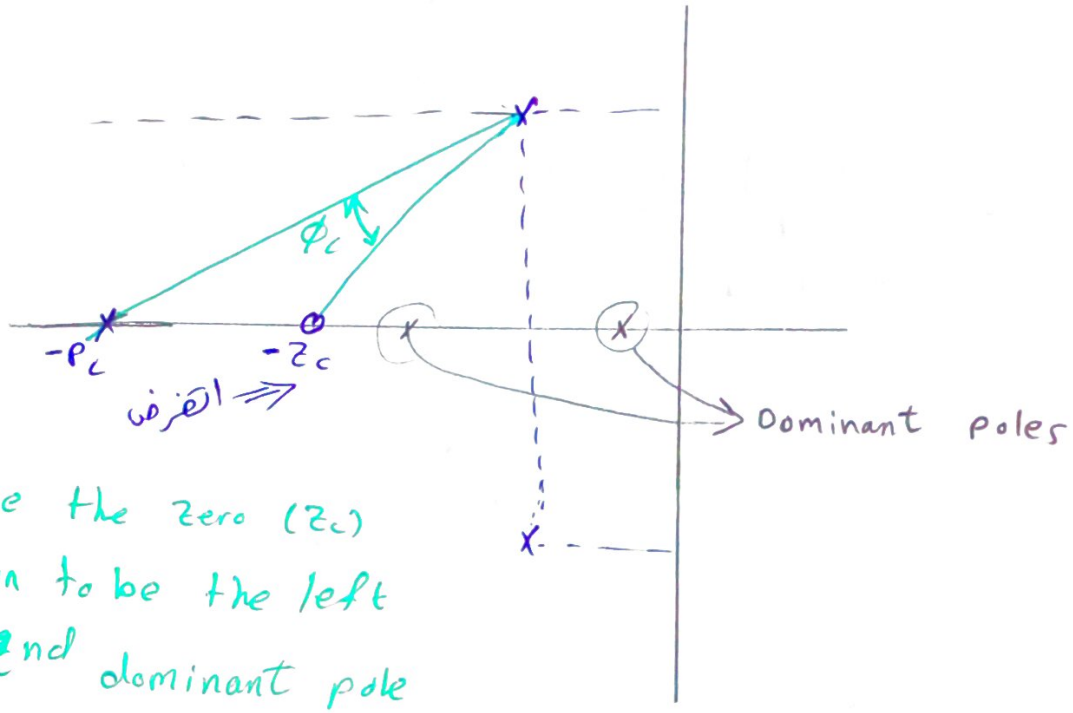


③ Determine the location of z_c. p_c by the known of ϕ_c

① Bisection Method



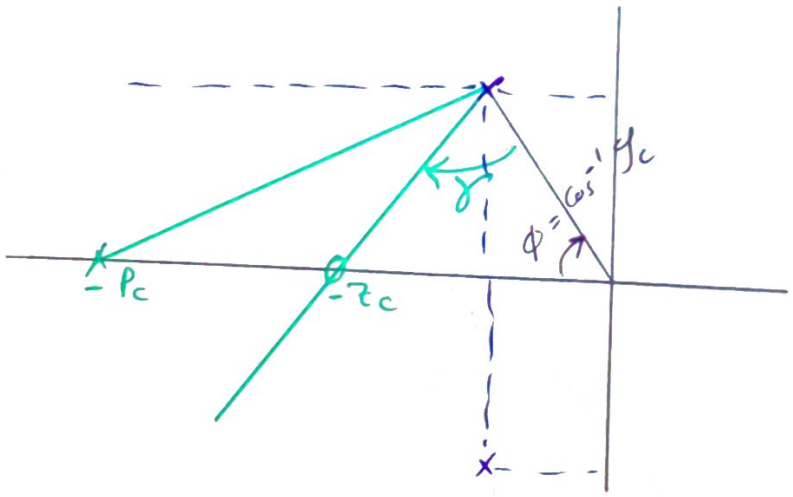
[2]



Assume the zero (z_c) location to be the left of 2nd dominant pole

[3] Maximum Attenuation ratio

$$\gamma = \frac{1}{2}\left[\pi - \phi - \phi_c\right] \qquad \phi = \cos^{-1}\zeta_c$$



[4] Apply the Magnitude Condition to determine the value of gain K to meet desired specs

$$\| K G_c(s) G H(s) \|_{s=s_d} = 1$$

$$K = \frac{\prod \text{Poles}}{\prod \text{zeros}}$$

Ex 1 : $GH(s) = \dfrac{K}{s(s+1)(s+4)}$

Design a compensator to meet the following specs: $\zeta = 0.5$ & $\omega_n = 2$ rad/sec, what is the type of controller ?

- as the desired specs are concered with system dynamics $\Rightarrow$ The controller is lead controller

① $s_{d_{1,2}} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ (the location of desired Poles)

$= -1 \pm j\sqrt{3}$

② Apply Angle condition to get $\phi_c = \phi_{z_c} - \phi_{p_c}$

$\angle GH + \phi_c = -180$

$\Downarrow$

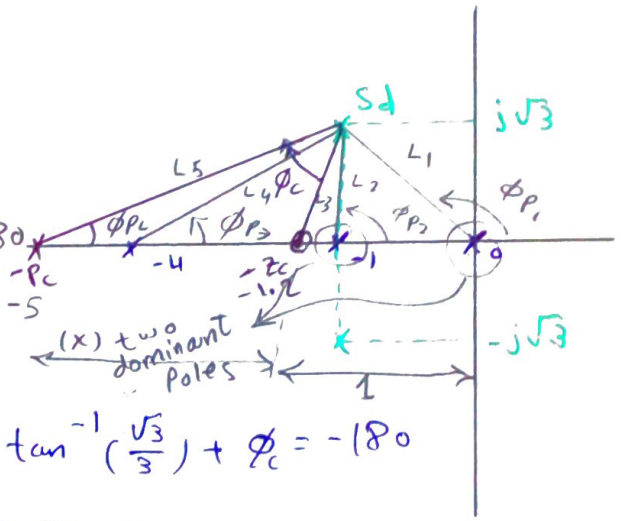
$[\phi_z - \phi_p]$

z زوايا ذات

p زوايا ذات

$-\phi_{p_1} - \phi_{p_2} - \phi_{p_3} + \phi_c = -180$

$\angle GH$



$\left[180 - \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)\right] - 90^\circ - \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) + \phi_c = -180$

$\phi_c = +60^\circ$ لذا نحتاج زاوية تعطي (+)

لأن الـ zero قبل الـ pole في حالة الـ lead

③ Determine the location of z_c, p_c :

assume $-z_c = -1.2$

(the left of 2nd dominant pole (-1)

$\Rightarrow$ from graph $-p_c = -5$

OR

$|p_c| = 1 + x \quad \Rightarrow \tan(\phi_{p_c}) = \dfrac{\sqrt{3}}{x}$

$\phi_c = \phi_{z_c} - \phi_{p_c}$

نعين

$$\phi_{2c} = \tan^{-1}\left(\frac{\sqrt{3}}{0.2}\right) = 83.413^\circ$$

$$GO = 83.413^\circ - \phi_{Pc} \Rightarrow \phi_{Pc} = 23.41^\circ$$

$$\tan(\phi_{Pc}) = \frac{\sqrt{3}}{x}$$

$$\tan(23.41) = \sqrt{3}/x$$

$$x = 4$$

$$|P_c| = 1 + 4 = 5$$

Total O.L.T.F $= K\, G_c(s)\, GH(s)$

$$= \frac{K(s + 2.2)}{s(s + P_c)(s+1)(s+4)}$$

[4] obtain K at $s_{d_{1,2}} = -1 \pm j\sqrt{3}$

$$\| K G_c(s) \cdot GH(s) \|_{s = s_d} = 1$$

$$K = \left\| \frac{s(s+5)(s+1)(s+4)}{(s+1.2)} \right\|_{s = -1 + j\sqrt{3}} = 30$$

OR

$$K = \frac{\prod \text{Poles}}{\prod \text{Zeros}} = \frac{L_1 L_2 L_4 L_5}{L_3}$$

[5] check steady state error (Useful for Lag compensator)

$r(t) = 1 \Rightarrow K_p = \lim_{s \to 0} GH(s) \longrightarrow$ s.s.e. $= e_{ss} = \frac{1}{1 + K_p}$

$r(t) = t \Rightarrow K_v = \lim_{s \to 0} s\, GH(s) \longrightarrow$ s.s.e $= e_{ss} = \frac{1}{K_v}$

$r(t) = t/2 \Rightarrow K_a = \lim s^2\, GH(s) \longrightarrow$ s.s.e $= e_{ss} = \frac{1}{K_a}$

$$K_v = \lim_{s \to 0} s\, K \cdot G_c(s) \cdot GH(s)$$

$$= \lim_{s \to 0} s \frac{30 (s + 1.2)}{s(s+1)(s+4)(s+5)}$$

$$= \frac{30 (1.2)}{(1)(4)(5)} = 1.8 \Rightarrow S.S.e = \frac{1}{K_v}$$

$$= \frac{1}{1.8} = 0.56$$